I Congresso Brasileiro de
Jovens Pesquisadores em
Matemática Pura e Aplicada

Singular perturbation methods for fully nonlinear degenerate equations: a geometric approach

Gleydson Chaves Ricarte*

*Universidade Federal do Ceará

Resumo

We establish new, optimal gradient continuity estimates for solutions to a class of 2nd order partial differential equations,

$$|\nabla u|^{\gamma} F(X, D^2u) = f,$$

whose diffusion properties (ellipticity) degenerate along the *a priori* unknown singular set of an existing solution, $S(u) := \{X : \nabla u(X) = 0\}$. The innovative feature of our main result concerns its optimality – the sharp, encoded smoothness effects of the operator. Such a quantitative information usually plays a decisive role in the analysis of a number of analytic and geometric problems.

For the second part, we study solutions of one phase singular degenerate singular perturbation problems of the type:

$$|\nabla u|^{\gamma} F(X, D^2u) = \beta_{\epsilon}(u),$$

where β_ϵ approaches Dirac δ_0 as $\varepsilon \to 0$. Uniform local Lipschitz regularity is obtained for these solutions. The existence theory for non variational (least supersolutions) solutions for this problem is developed. Uniform linear growth rate with respect to the distance from the ε-level surfaces are established for this nonvariational solutions. Finally, letting $\epsilon \to 0$ basic properties such as local Lipschitz regularity and non-degeneracy property are proven for the limit and a Hausdorff measure estimate for its free boundary is obtained. An important question on modern theory of elliptic free boundaries problems concerns whether is possible extend the local optimal regularity of singularly perturbed problems up to the boundary. In this article, under appropriate conditions on the operator F we study regularity up to the boundary for one-phase singularly perturbed fully nonlinear elliptic problems,

$$F(X, \nabla u^{\varepsilon}, D^2u^{\varepsilon}) = \beta_{\varepsilon}(u^{\varepsilon}), \quad \text{in } \Omega$$

we establish global gradient bounds independent of the parameter ε.